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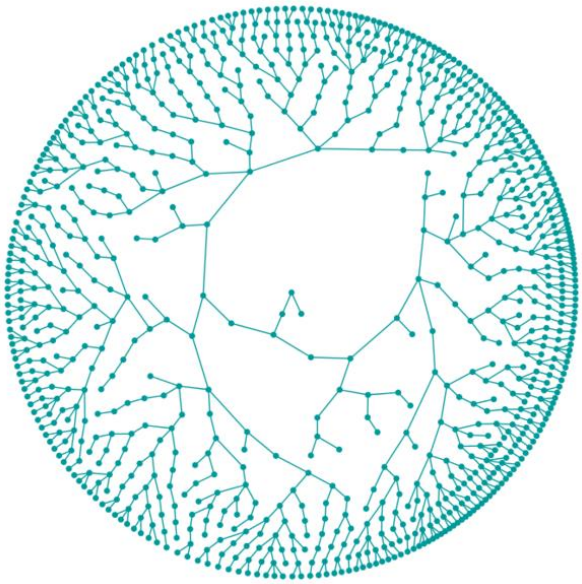

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Content

A new class of distribution for variable selection in zero-inflated longitudinal count data

Alsalim, N.*, and Baghfalaki, T.

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Abstract: In modeling many longitudinal count clinical studies, the excess of zeros is a common problem. To take into account the extra zeros, the zero-inflated power series (ZIPS) models have been applied. These models assume a latent mixture model consisting of a count component and a degenerated zero component that has a unit point mass at zero. In this paper, we use transition models for modeling longitudinal data. Also, new variable selection methods for the ZIPS transition model using least absolute shrinkage and selection operator (LASSO), minimax concave penalty (MCP) and smoothly clipped absolute deviation (SCAD) penalties are proposed. An expectation-maximization (EM) algorithm using the penalized likelihood is applied for both parameters estimations and conducting variable selection. The approach is applied to analyze a real data set.

Keywords: The EM algorithm, Longitudinal data, Penalized likelihood, Transition model, Variable selection.

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Semi-parametric generalized additive regression with nonparametric rank reduction

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Abstract: The rank reduction is a technique for decreasing the number of parameters in a model. The nonparametric rank reduction is used to reduce the number of free coefficients of a spline estimator of the smooth function. In this paper, a rank reduction technique is introduced in a semiparametric regression model along with the SCAD penalities for the linear and nonlinear parts to perform a vaiable selection, simultaneously. The result of a simulation study shows that the proposed semi-parametric generalized linear reduced rank model performs better than the full-nonparametric reduced rank generalized additive model.

Keywords: *k*-nearest neighbour regression, logistic regression, partial linear model, spline smoothing.

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Derivation of order conditions for a family of split-drift stochastic Runge-Kutta schemes

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Abstract: In this paper we extract the order conditions for a family of split-drift stochastic Runge-Kutta (SDSRK) schemes for systems of Ito stochastic differential equations (SDEs). The order conditions of the desired stochastic weak order are obtained with a modified analysis of colored rooted trees. Then, for a subclass of weak second order SDSRK methods, corresponding order conditions will be obtained.

Keywords: Stochastic differential equations, Stochastic Runge-Kutta schemes, Split-drift SRK.

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An application of stochastic birth/birth-death processes for production system maintenance

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Abstract: This work considers the birth/birth-death process, a tractable bivariate extension of the birth-death process, in order to model the state transition of a multi-unit deteriorating system. In such a system, each unit is subject to gradual deterioration and the deterioration process of each unit is a three-state continuous time-homogeneous Markov chain with two working states and a failure state. Our interest is to obtain the first passage time distribution and the mean first passage time to a set of system taboo states with a pre-specified number of failed units. Finally, a numerical example of a multi-unit wind farm system is being used to demonstrate the applicability of the provided method.

Keywords: Birth/Birth-death processes, Mean first passage Time, Multi-unit system, Maintenance.

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Stochastic comparisons of parallel systems with Topp Leone Kumaraswamy-G components having Archimedean copulas

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Abstract: This paper treats the problem of stochastic comparisons of two parallel systems with dependent heterogeneous components having lifetimes following Topp Leone Kumaraswamy-G model. We obtain the usual stochastic order for the largest order statistic of samples having Topp Leone Kumaraswamy-G model and Archimedean copulas. Some examples are provided to illustrate the established results.

Keywords: Archimedean copula, Topp Leone Kumaraswamy-G model, Majorization, Usual stochastic order, Parallel systems.

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Galton-Watson branching process

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Abstract: Galton-Watson process was introduced by Francis Galton as a simple mathematical model for the propagation of family names. In this paper, we review this process and its applications in various fields. Finally, using Monte Carlo simulation, we evaluate extinction probability for some specific choices of the offspring distribution.

Keywords: Galton-Watson process, Extinction probability, Offspring distribution.

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A maximal probability inequality in Riesz spaces

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Abstract: We establish a maximal probability inequality in the framework of Riesz spaces.

Keywords: Riesz space, Kolmogorov inequality, Weak order unit, Band projection.

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On diagnostic checking for generalized first-order integer-valued autoregressive process

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Abstract: In recent years, integer-valued time series attract the attention of researchers and find their applications in data analysis. Among various models, the integer-valued autoregressive (INAR) ones are of great popularity and are widely applied in practice. This paper develops some portmanteau test statistics to check the adequacy of the fitted model in a wide group of INAR processes, called generalized integer-valued autoregressive (GINAR). For this purpose, the asymptotic distributions of the test statistics are obtained and, using Monte Carlo simulation studies, their finite sample properties are derived. Besides, the results are applied in analyzing a real data example.

Keywords: Integer-valued time series, Generalized INAR, Portmanteau tests, Asymptotic distribution.

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A stochastic modeling of toroidal data

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Abstract: Despite much progress in developing stochastic processes in the Euclidean spaces, few activities were devoted to proposing those for the manifold-valued data. These typical data, taking their values on a manifold, are observed in many scientific disciplines including biological sciences. The problem will be much harder if the manifold-valued data inherit some specific periodic properties. We propose an specific model to tackle the perturbations of the angles taking their values on the surface of a torus. We first construct the relevant tensor metric formed on the standard torus by particular vector fields and then use it to derive the stochastic differential equations for the Brownian motion and Ornstein-Uhlenbeck processes. After deriving the stationary distributions of the corresponding diffusion processes, we also propose some related statistical inferences. Extensive simulation studies are then conducted to evaluate the proposed models. Analyzing a real-life example is also done to highlight an application of the idea.

Keywords: Diffusion process, Stochastic differential equation, Manifold-valued data, Torus, Von Mises density.

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On two new systems: Probability structures and applications

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Abstract: In this paper, two new systems are introduced where both systems could be widely used in various fields of engineering, economics, and medicine. Since any given system can be represented as a series arrangement of parallel structures or a parallel arrangement of series structures, we introduce two extensions of these systems with a random number of components in their series and parallel structures. Their probability Structures are discussed in this paper. We also derive formal expressions for their cumulative distribution functions and probability density functions. Finally, an application of the models with a real dataset is presented to illustrate the usefulness of the proposed systems.

Keywords: Compound distribution, Cumulative distribution function, Maximum likelihood estimation, Parallel-series system, Series-parallel system.

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A simple expression for the Fisher information matrix of a normal mixture distribution

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Abstract: The Fisher information matrix for a mixture of two normal distributions is derived. The explicit expression for the elements of the matrix using the Fourier transformation are obtained. Numerical results of the matrix are prepared for practical purposes in several tables.

Keywords: Fisher information matrix, Fourier transform, Normal distribution.

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The optimal design for one-shot devices under step stress accelerated life tests

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Abstract: A one-shot device, like an automobile airbag, is a product or an equipment that can be used only once. Better quality and longer lifetime of one-shot devices nowadays increase the cost of life test experiment under normal operating condition. Step stress test, adopted in life-testing experiments by increasing stress levels to induce more failures, has been used to investigate the reliability analysis of one-shot devices. At each stress level, a exponential lifetime distribution is considered. The scale parameter of the proposed distribution is assumed to be a log-linear function of stresses. To conduct a one-shot device accelerated life test more efficiently, one has to address the problem of determining optimal setting that produces the best estimation results. This paper provides the optimal design of simple step-stress accelerated life tests for one-shot devices by minimizing the asymptotic variance of the maximum likelihood estimate of reliability at normal operating conditions under exponential distribution, with respect to inspection times. Simulation study is carried out to study the optimal design. Besides, sensitivity analysis is conducted to evaluate the robustness of determined designs when mis-specification on planning values of model parameters exists.

Keywords: Exponential distribution, one-shot devices, optimal design, step stress accelerated life tests.

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Functional multifractal detrended fluctuation analysis

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Abstract: In this work, we generalize the multifractal detrended fluctuation analysis (MDFA) to the Functional case, named Functional MDFA (FMDFA). The validity of the proposed FMDFA is illustrated by numerical simulations.

Keywords: Long memory, Detrended fluctuation analysis, Functional time series.

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Population-structured Bayesian epistasis association mapping

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Abstract: Association mapping is an important field of research in bioinformatics and human genomics studies. Various statistical methods have been used to find out genetic regions is associated with traits or phenotypes. One of the methods used in the association mapping is clustering. In this paper, we introduce a clustering method with special application in association mapping. In this method, the inference of disease associated factors of each sub-population is simultaneously performed by clustering assignment of individuals, based on similarity of genetic samples. A framework is proposed to approximate the posterior of the model based on, either Markov Chain Monte Carlo (MCMC), or Variational Bayes (VB) methods, and Bayesian Epistasis Association Mapping (BEAM) model for disease association discovery.

Keywords: Association mapping in Genome-Wide Association Studies, Disease graph, Single Nucleotide Polymorphism interactions.

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Spatial shot-noise Cox processes with elliptically symmetric kernels

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Abstract: Data in the form of clustered spatial point patterns are commonly encountered in different applications. Shot-noise Cox point processes are among suitable probabilistic models for such data. In the present work, we discuss spatial shot-noise Cox processes with elliptically symmetric kernels and suggest the generalized Gamma kernel as a flexible model for describing various types of cluster structures. In addition, we drive a closed formed expression for the pair correlation and K functions of shot-noise Cox processes with elliptically symmetric kernels. Particularly, we show that the pair correlations of a shot-noise Cox process with the generalized Gamma kernel exhibits different shape and tail behaviors compared to the pair correlation function of the shot-noise Cox processes with the usual kernels.

Keywords: Cluster point process, Pair correlation function, K function, Generalized gamma kernel, Gauss' hypergeometric function.

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A new two-sided lifetime distribution: Some inference based on frequentist and Bayesian approaches

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Abstract: In present paper, we define a new two-sided probability distribution. The new model is called the general two-sided log logistic exponential distribution. We estimate the parameters of the proposed distribution by maximum likelihood and Bayesian methods. We show that the proposed model fits better than the generalized two-sided exponential and generalized transmuted two-sided exponential distributions.

Keywords: Bayesian method, Maximum likelihood, Hazard rate function, Two-sided distribution.

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Markovian analysis of semi-Markov queues

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Abstract: This article concerns with some typical analysis of non-Markovian (semi-Markovian) queueing systems. Strictly talking, we will consider those systems which are Markovian in one main part; arrival or departures, and is non Markov in the other part; departure or arrival, respectively. Since only one part of the queueing system has the Markov property, it is called semi-Markov and realy it is (semi-Markov). By the fact that the class of these systems is very large and includes a wide range of queueing models, we only practice the simplest cases, M/G/1 and G/M/1. The PASTA property of Poisson processes is the most essential tool.

Keywords: Markov chains, PASTA property, Poisson processes, Queueing systems.

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A quick review of the shock models: Properties, relations and applications

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Abstract: The most important types of shock models are δ-shock, extreme shock models and mixed of them. In this paper, we will have a quick review of these models and their applications. A short comparison between these models is also provided.

Keywords: δ-shock model, Extreme shock model, Mixed shock model.

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A mixture of covariated time series process with application to COVID-19

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Abstract: As the COVID-19 pandemic spread worldwide, it has become clearer that prevalence of certain comorbidities in a given population could make it more vulnerable to serious outcomes of that disease, including fatality. Indeed, it might be insightful from a health policy perspective to identify clusters of populations in terms of the associations between their prevalent comorbidities and the observed COVID-19 specific death rates. In this study, we described a mixture of polynomial time series (MoPTS) model to simultaneously identify (a) three clusters of 86 U.S. cities in terms of their dynamic death rates, and (b) the different associations of those rates with 5 key comorbidities among the populations in the clusters. We also described an EM algorithm for efficient maximum likelihood estimation of the model parameters.

Keywords: Mixture of regressions; EM algorithm; Death rate; Comorbidities; COVID-19.

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Discrete prolate spheroidal sequences-based probability density estimation for non-stationary data

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Abstract: In this paper, we consider probability density function (PDF) estimation of data which arrived in time-varying fashion such as computer network traffic, ATM transactions, web searches. For such situations, the consecutive data are correlated or the data arrives along time and we cannot store all the data in one static database. So, the traditional PDF estimators are not applicable. Thus, we propose a new PDF estimation algorithm that is based on Discrete Prolate Spheroidal Sequences (DPSS). The key innovation behind this approach is projecting the non-stationary processes on the orthonormal DPSS and using a sliding window for adaptation of the non-stationarity feature of such processes. The probability density estimation is achieved by estimating the PDF of the uncorrelated expansion coefficients. The performance and accuracy of the estimation algorithm is investigated by means of simulations and some numerical experiments were carried out. The proposed estimator is compared to the Kernel estimator. In all cases, the DPSS-based estimator yields more accurate results, which confirmed high efficiency of the proposed method.

Keywords: Non-stationary processes, Probability density estimation, Discrete prolate spheroidal sequences.

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The skew Laplace normal periodic autoregressive time series

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Abstract: We consider here a periodic autoregressive model with skew Laplace normal innovations and our aim is to estimate the model parameters. The maximum likelihood estimation method is then developed by using the expectation–conditional maximization algorithms. The numerical results obtained by means of simulation studies are also reported.

Keywords: Periodic autoregressive models, Skew Laplace normal, VAR models, ECM algorithms.

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On comparing some numerical estimation methods of VAR(1) models with MGST innovations

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Abstract: A vector autoregressive model of order one with multivariate generalized scaled t-distributed innovations is considered here. The object is to estimate the parameters of the proposed model by using the well-known maximum likelihood estimation method. The maximum likelihood estimation method is performed by using the expectation–conditional maximization algorithms (ECM and ECME) to accelerate the basic EM algorithm. The proposed methods are also compared to a quasi-Newton well-known method, named BHHH in terms of the rate of convergence as well as the precision and validity of the obtained estimates, by implementing some numerical simulations.

Keywords: Autoregressive models, Multivariate generalized scaled t distribution, VAR models, ECM algorithm, ECME algorithm, BHHH algorithm.

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A new family of spherical distribution to model asymmetric spherical data

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Abstract: The Pearson type family densities are among the most important classes of distributions, playing also key roles in directional statistics. To model data scattered asymmetrically on non-Euclidean spaces including spheres, the researchers confined themselves to extend particular distributions from the class of the Pearson type family densities. Those specific distributions are symmetric in nature but their extended versions are usually heavy-tailed. In this paper, we introduce some alternative probability density functions in the class of Pearson type distributions on the sphere having the spherical Student's t, Fisher and Chi-square densities as the subfamilies. Via investigating various theoretical properties of this new subclass, we show that it is intrinsically asymmetric. Also, modeling two real-life data using the proposed densities and then comparing the results with the fits from other common spherical distributions are considered.

Keywords: Spherical densities, Pearson type family, Heavy-tailed distributions, Asymmetric distributions.

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Copula parameter estimation using nonlinear quantile regression

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Abstract: The aim of this paper is to propose a semiparametric method for the estimation of the copula parameters based on a nonlinear quantile regression model. The estimation of the dependence parameter has been selected as the value that minimizes the distance between one of the pseudo samples and the inverse of the quantile regression. A simulation study is performed to measure the performance of this method. The simulation results are compared to the maximum pseudo-likelihood (MPL) method and minimum pseudo-Hellinger distance (MPHD) method for well-known bivariate copula models. These results show that the proposed method based on the copula quantile regression model has a good performance in small sample sizes.

Keywords: Quantile regression, Copula parameter estimation, Minimum distance, Semiparametric Estimation.

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Integer valued AR(1) process with zero-and-one inflated geometric innovations

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Abstract: In this paper, zero and one inflated Geometric distribution is introduced and some basic properties of it are obtained. The first order integer valued autoregressive model with zero and one inflated Geometric distributed innovations is presented. Several statistical properties of the process are calculated. Estimation of the model parameters are obtained by the conditional least square and conditional maximum likelihood methods. The performance of the estimation methods is checked by a small Monte Carlo simulation.

Keywords: INAR process, Geometric distribution, Zero and one inflated Geometric distribution.

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Some functional linear regression models for interval-valued data

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Abstract: The interval-valued functional data belongs to the category of data in which the values of each observation are consisted of two functions, a lower, and an upper limit function. The current study introduces some approaches to deal with a functional response and functional predictor linear regression models focusing on interval-valued data. These models are based on univariate and bivariate basis functions, use leave-one-curve-out cross-validation to extract the cut-off point of the number of bases. Also, the parameters are estimated via a least-square estimate method. The first two proposed methods analyzes the interval data based on two independent linear regression models, which cannot be actualized in general. A solution might be the combination of the two regression models which are illustrated in the other proposed methods.

Keywords: Cross-validation, Functional data analysis, Functional linear model, Interval-valued data.

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Strong convergence of weighted sums of APND fuzzy random variables

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Abstract: In this paper, we give some conditions for a.s. convergence of weighted sums of APND integrable fuzzy random variables. As a result, we obtain strong laws of large numbers for weighted sums of independent fuzzy random variables.

Keywords: Fuzzy random variables, Strong laws of large number, Weighted sums.

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Poisson-Kingman distribution generated by negative binomial processes

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Abstract: We study a new class of random discrete distributions on the infinite unit simplex, and in particular a subclass which is a generalisation of the Poisson-Kingman class of distributions. The new classes are based on a negative binomial rather than a Poisson point process. We derive formulae for distributions relating to size-biased sampling and the corresponding EPPF (exchangeable partition probability function). The distributions are fitted to some species count data.

Keywords: Poisson-Kingman distribution, exchangeable partition probability function (EPPF), negative binomial point process, trimmed α-stable subordinator.

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A real data analysis for autoregressions driven by generalized skewed Linnik distributions

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Abstract: Heavy tail distributions are great interest over the last few years in science, economics and industrial and have proved to be strong alternatives to the Gaussian distribution. There have been several works in the literature introducing and discussing on the Pakes generalized Linnik distribution and its relative distributions. However, most of these works consider only the special case of symmetric random variables. This is an important restriction though, since most real life data are skewed. In this paper, we discuss a new class of generalized skewed Linnik distribution and its Autoregressive model. A real data analysis for such distributions is presented and to show how strong this method is, the model is considered and fitted to a set of time series data.

Keywords: Autoregressive process, Geometric stable distributions, skewed Linnik distribution, Time series.

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Estimation and model selection in GARCH model

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Abstract: The generalized autoregressive conditional heteroskedasticity process is an econometric term and describes an approach to estimate volatility in financial markets. In this paper we consider general autoregressive conditional heteroscedastic model, where innovation terms follow exponential family which contains symmetry (such as normal distribution) and asymmetric distribution. One of the problems for these models is parameter estimation. The maximum likelihood estimators are unsuccessful in some situations since explicit solutions from the likelihood equations cannot be obtained. Here, we estimate the parameters of model based on the modified maximum likelihood method. So that, we can select optimal model based on the Vuong's test. Our results of simulation study show that the performance of modified maximum likelihood estimators is better than maximum likelihood estimators. We have studied real data, the CBOE Equity VIX on Apple data, and select optimal model for these data based on the theoretical results.

Keywords: Financial data, General autoregressive conditional heteroscedastic model, Model selection, Modified maximum likelihood, Vuong's test.

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